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Fragmentation in Simple Trade Models

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ABSTRACT

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This paper examines the effects of "fragmentation," defined as the splitting of a production process into two or more steps that can be undertaken in different locations but that lead to the same final product. Introducing the possibility of fragmentation into simple theoretical models of international trade, the paper finds the effects of fragmentation on national welfare, on patterns of specialization and trade, and on factor prices. Models examined include the Ricardian Model and the Heckscher-Ohlin Model, both for small open economies and for a two-country world. Results are as follows: 1. If fragmentation does not change the prices of goods, then it must increase the value of output of any country where it occurs and that of the world. 2. If fragmentation does change prices, then fragmentation can lower the welfare of a country by turning its terms of trade against it. 3. Even in a country that gains from fragmentation, it is possible (but not necessary) that some factor owners within that country will lose. 4. To the extent that factor prices are not equalized internationally in the absence of fragmentation, fragmentation may be a force toward factor price equalization.

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Fragmentation in Simple Trade Models [*]

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I. Introduction

The subject is fragmentation: the splitting of a production process into two or more steps that can be undertaken in different locations but that lead to the same final product. Also called "intra-product specialization" by Arndt (1996) and by the more loaded term "outsourcing" in some economic literature as well as in the popular press, fragmentation occurs both within countries and across countries.[1] Within countries, if domestic factor markets are well integrated and markets are competitive, then fragmentation would be expected to occur only if the combined resources used by the fragmented steps were less than those used by the original process, in which case fragmentation would also represent a technological improvement. In this paper I will assume instead that fragmentation does not economize on resources, and therefore I will focus on fragmentation that occurs across countries.[2]

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[*] I have benefited from discussions on this topic with Peter Debaere, Sven Arndt, and other participants in the Globalization workshop held at Claremont in January 1997, as well as participants in the NAEFA session for which the paper was initially written, including my discussant, Matthew Slaughter

[1] The term "fragmentation" comes from Jones and Kierzkowski (1997). Several other terms for this phenomenon were also suggested at the NAEFA session, none seeming to generate a consensus.

[2] Even without any saving in resources, fragmentation could occur within a country for a variety of reasons. Perhaps the most obvious is if labor markets are not competitive, so that wages are bid up in some sectors by the bargaining power of unions. Fragmentation in order to divert some parts of a production process outside the purview of the union then makes obvious economic sense. Even with perfect competition, if a country is "lumpy" – that is, if different equilibrium wages are paid in different regions within the country as in Courant and Deardorff (1992) – then fragmentation may occur across regions for much the same reasons as the international fragmentation I will examine here.

Internationally, fragmentation has become increasingly common in recent years as barriers to international trade and investment have fallen and as an increasingly competitive world environment has forced producers to look outside their own borders for ways to reduce costs. In the debate over the causes of increased wage inequality in the United States in the 1980s and 90s, "globalization" and technology have both been suggested as important causes of the increased wage differential paid to skilled labor, globalization being represented variously by international trade, foreign direct investment, factor mobility, and outsourcing.[3] In fact, fragmentation may be thought of as a manifestation of globalization and technology combined, since in many industries it is only advances in technology that have made the splitting of production processes and the coordination of the resulting parts possible.

In any case, with the exception of Arndt (1996) and Jones and Kierzkowski (1997), the economic effects of fragmentation do not seem to have been given the theoretical treatment they deserve, and in this paper I will attempt to correct that.[4] Using several familiar and simple models of international trade, I will examine the implications of fragmentation on trade, patterns of specialization, and factor markets, looking especially at its effects on factor prices and on the overall welfare of the countries involved.

I will examine the effects of fragmentation first in a Ricardian model in section II, then in a Heckscher-Ohlin model in section III. Section IV concludes.

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[3] There have been many surveys of this voluminous "trade and wages" literature, most recently by Johnson and Stafford (1997). One pair of papers that has focused attention on outsourcing is Feenstra and Hanson (1996, 1997).

[4] The literature now also includes two of the other papers prepared for the NAEFA session for which this was written. Not coincidentally, they are by the same authors: Arndt (1997) and another paper by Jones

II. Fragmentation in Ricardo

I will look first at the effects of fragmentation on a small open economy, then at a large country in a two-country world.

A Small Open Ricardian Economy

Consider first the textbook case of only two goods. The country is endowed with a fixed amount of labor, L, and it can use this labor to produce either of two (final) goods, X and Y. The unit labor requirements of each are fixed at a_X and a_Y respectively. As a small open economy, the country faces fixed prices of the two goods on a world market, p_X and p_Y, at which it can buy or sell unlimited quantities. As usual, if the wage in the country were w, then the supply prices of the two goods would be wa_X and wa_Y respectively, and profits would be made if either of these were below the corresponding world price. Therefore the market equilibrium wage is instead

$$w_0 = \max\left(\frac{p_X}{a_X}, \frac{p_Y}{a_Y}\right) \tag{1}$$

The country specializes in whichever good yields this highest wage, exporting X if $p_X/a_X > p_Y/a_Y$ and vice versa. Equivalently, if

$$\frac{a_X}{a_Y} < \frac{p_X}{p_Y}, \tag{2}$$

then the country has a comparative advantage in X and will export it. I will assume this to be the case.

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and Kierzkowski for which I do not have the exact citation. The empirical literature on fragmentation has also been expanded with a new paper by Slaughter that I also don't have.

Now suppose that fragmentation becomes possible in the X industry. In general, this means that the process for producing one unit of good X can be split into multiple parts, i=1,…,n_X, each requiring an amount of labor a_{Xi}, and since I do not want to conflate fragmentation with technological progress, I assume that $\sum_{i=1}^{n_X} a_{Xi} \geq a_X$. Coordination of these fragmented parts might in general require oversight by a single entity such as a multinational enterprise (whose function could itself be represented by one of the parts), but to consider explicitly how the parts are organized would take me too far afield.[5] Instead I will define each part as producing one unit of an intermediate good, Z_{xi}, that is also an intermediate input to the next part that produces one unit of Z_{xi+1}, the final step producing a unit of good X itself: $Z_{Xn_X} = X$. Thus production of a unit of each intermediate input except the first requires, in addition to the labor a_{Xi}, one unit of the preceding intermediate good, Z_{xi-1}.

In general not all of these intermediate goods need be regarded as tradable; their tradability indicates whether two adjacent parts of the fragmented production process can take place in different countries. However, if they are not tradable, then their separate identities will play no role in the model here with an integrated competitive domestic labor market, and they could be combined into one. Therefore, I will assume that all of these intermediate goods are internationally tradable.

What I have described is a fairly general formulation of fragmentation, but in the spirit of my initial 2-good model, let me start with the simple case of n_X=2. That is, let

[5] The analysis here is general enough, both in this section and the next, to include the sorts of disembodied contributions to a firm's operations that Helpman (1984) took to be the hallmark of a multinational corporation. Although I will define each fragmented part of the production process as producing an

production of X be fragmented into just two parts, requiring amounts of labor a_{X1} and a_{X2}, the first of which produces an intermediate good Z, that is required one-for-one with production of good X in the second part. The result is simply a 3-good Ricardian model, where the three goods are X, Y, and Z, and where the demand for Z is the quantity of X that is produced using the fragmented technology. Since Z is tradable, introduction of fragmentation also requires that the world market provide an additional price for it, p_Z. The question is, what does all of this do to the small open economy facing this and the other prices?

The answer depends in part on whether those other prices – p_X and p_Y – also change as a consequence of fragmentation. This is an issue that will come up more clearly in the 2-country model below, so for now I simply assume that they do not. This will in fact be the case if the large rest-of-world with which our small country trades is itself a fully integrated economy. In that case, there is a single wage of labor prevailing throughout the rest-of-world, and fragmentation that does not lower total labor requirements cannot lower the price of X. Its price cannot rise, either, since the original unfragmented technology is still available.

What matters for the small country, then, is the price of Z, which will depend on the parameters of the fragmented technology in the rest-of-world. Since this is a Ricardian model, these are not necessarily the same as in the small country, and I will therefore just examine how behavior in the small country depends on p_Z.

The answer is simple. Labor now has two additional options for employment, in addition to producing X from scratch and producing Y. It can produce Z, earning a wage

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intermediate input that is then used in the next stage of production, this intermediate input can be a fiction,

p_Z/a_{X1}, or it can produce X from Z, earning a wage $(p_X - p_Z)/a_{X2}$. Labor will therefore engage in whichever of the four activities earns it the most, and it will do more than one thing only if two or more of the activities yield the same wage. Thus the equilibrium wage in the fragmented small open economy, w_F, is

$$w_F = \max\left(\frac{p_X}{a_X}, \frac{p_Y}{a_Y}, \frac{p_Z}{a_{X1}}, \frac{(p_X - p_Z)}{a_{X2}}\right) \tag{3}$$

The country will engage in whichever activity or activities yields this maximum wage.

The country's production and trade are summarized in Figure 1. By assumption (2) that it has a comparative advantage in good X, it will never produce good Y. Whether and how it produces good X, however, depends on the price of the intermediate input. Taking good X as numeraire, comparison of the arguments in (3) shows that if p_Z is sufficiently low – below $\frac{a_X - a_{X2}}{a_X}$ – then the country will employ only the second fragmented part of the new technology for producing X, importing all it needs of good Z from the rest-of-world. For a somewhat higher price of the intermediate input, however – above $\frac{a_X - a_{X2}}{a_X}$ but below $\frac{a_{X1}}{a_X}$ – it will continue to produce X from scratch with the old technology. Finally, if the price of the intermediate is high enough – above $\frac{a_{X1}}{a_X}$ – then it will produce only Z. In the latter case, since it does not produce X and therefore does not need Z, it exports all of its output of Z. The figure shows these responses of output to

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at least until one attempts to apply this approach to the data.

price as the heavy solid line for *X* and the heavy dashed line for *Z*. In addition, it shows net trade in *Z* (exports if positive, imports if negative) as the heavy dotted line.[6]

For some prices, then, Figure 1 shows the fragmented technology not being used at all. This occurs only if $\frac{a_{X1}}{a_X}$ is in fact greater than $\frac{a_X - a_{X2}}{a_X}$, which it may not be. This requires that

$$a_{X1} + a_{X2} > a_X , \tag{4}$$

or in other words that the fragmented technology use more resources than the original, or that fragmentation is, in this sense, "costly." I will take this to be the normal case in this section, although the justification for doing so most easily relies on transportation costs, which I am assuming away in the rest of the model.

In any case, the model says that if fragmentation is costly, then for some parameters and prices it will not be used, but for other parameters it will. Also, if fragmentation is not costly, then the middle range of behavior in Figure 1 collapses to a single price, and fragmentation is used at all prices but one, $\frac{a_{X1}}{a_X}$, where all are indifferent between producing *Z*, producing *X* from scratch, and producing *X* from *Z*.

It might seem in the case of costly fragmentation that welfare might fall, since the country is using part of what seems to be an inefficient technology. In fact, however, we see immediately from (3) that welfare cannot fall with the introduction of fragmentation

[6] Trade in *X* and *Y* are not shown, since they depend on preferences and income. Good *Y*, which is not produced, must be imported for all p_Z, although the quantity imported will be somewhat larger when the fragmented technology is being used, due to the country's higher income, and it will fall with p_Z for low p_Z and rise with p_Z for high p_Z due to changes in the terms of trade in *Z*. When only good *X* is produced, then part of it must be exported in exchange for Y, while it must be imported when it is not produced. Again, the quantities kept or imported depend on real income, which is larger when the fragmented technology is being employed and depends then on the p_Z.

in this Ricardian Model. The entire population is labor, and everyone earns a common wage that is at least as great as it was before, in real terms since prices of consumer goods are fixed. The reason, of course, is that while using both parts of the fragmented technology would indeed be inefficient, the country does not do that. On the contrary, it uses only one part of it, and the other part is not used anywhere. It is the latter part that is really inefficient at prevailing prices.

So far I have assumed that fragmentation occurs only in the export industry. That, of course, need not be the case. If fragmentation were to become possible in producing good *Y* instead, then the above analysis would be modified but only slightly. Parameters a_{Y1} and a_{Y2} would replace their *X* counterparts, and *Y* would replace *X* in the third and fourth arguments of (3). The result is again a range of prices of the intermediate good *Z* for which fragmentation is not observed, although in this case it will continue to be good *X*, not the potentially fragmented good *Y*, that is produced in the mid-range.[7] If the price of the intermediate is outside this range, however, good *X*, will not be produced at all, and the country will either produce and export the intermediate or produce and export *Y*. Thus fragmentation may give a country a comparative advantage in a good where it had no comparative advantage before.

This is about all that the simple Ricardian model of a small economy can tell us, but the results are pretty clearly robust to expansion of the model to include, say, more goods or more degrees of fragmentation. With both of these arbitrarily large, the wage is set as

$$w_F = \max_{\substack{j=1,\ldots,n \\ i_j=1,\ldots,n_j}} \left(\frac{p_j}{a_j}, \frac{\left(p_{ji_j} - p_{ji_j-1}\right)}{a_{ji_j}} \right) \tag{5}$$

where the technology for producing each of n goods j is potentially fragmented into n_j parts, the last of which produces the final good itself. p_j, a_j are the prices and unit labor requirements for producing the j^{th} good, $j=1,\ldots,n$, using the original Ricardian technology. p_{ji_j}, a_{ji_j} are the prices and unit labor requirements for the i_j^{th} intermediate input to good j, $i_j=1,\ldots,n_j$, with $p_{j0}=0$ and $p_{jn_j} = p_j$.

From this, it follows that the country will tend to specialize in using only one fragment of one technology, unless parameters and prices coincidentally align so that more than one process yields the same wage. Comparative advantage now refers most appropriately to processes, not goods, although we may identify them with the intermediate goods that they produce. As in the 2-good model, as long as prices of final goods in the rest-of-world are given and unchanged by the introduction of fragmentation, the small country cannot lose from it. Patterns of production and trade will not be determined quite as simply as in the 2-good model, but one of the patterns suggested in Figure 1 will continue to hold: given the prices of all other goods and fragments, a particular fragment will be employed and its intermediate product exported if its price is high enough. A low price, on the other hand, will not necessarily assure that it will be imported, since that also requires a high price in the next stage of production.

[7] This mid-range will exist even if fragmentation in the import-competing good is costless, since fragmented production must now overcome the comparative disadvantage in (2).

A Two Country Ricardian World

A small country in the Ricardian model, then, cannot lose from fragmentation so long as prices of final goods remain fixed. And fixity of prices has a certain plausibility if the rest-of-world is integrated, as noted above. However a large country can surely not take prices as given, and the problems that arise for a large country are also those that arise for a non-integrated rest-of-world. We therefore need to look at a two-country world in order to explore these problems. In that context, I will focus only on one issue: whether a country can lose from fragmentation. Along the way we will incidentally see a bit more about how fragmentation affects specialization and trade.

For a country to lose from fragmentation, prices must turn against it, and this requires that it be trading initially so that there is scope for its terms of trade worsen. I will consider, therefore, a country that specializes initially in good *X*, and try to see whether fragmentation can lower the relative price of good *X* sufficiently for it to lose. I will consider the same simple form of fragmentation that I looked at before, splitting production of good *X* into two parts. For simplicity and to avoid the costs of fragmentation itself being the source of any loss, I will now assume that fragmentation is costless. Analogous other cases should not be hard to find.

Figure 2 tells us most of what we need to know, although some explanation is required. In the upper left corner is drawn the production possibility frontier (PPF) for the country of interest, Country A, which is assumed again to have a comparative advantage in good *X*. Its customary Ricardian transformation curve is T_AT_A' in the *X*-*Y* plane, showing the maximum amounts of goods *X* and *Y* that it can produce without fragmentation. Fragmentation expands its production possibilities into a third dimension,

with the Z axis measuring its net output of intermediate good Z, positive if it produces it, negative if it is a net user of Z in production of X. Point Z_A shows the maximum amount that can be produced if all labor is devoted to production of good Z. Production possibilities include the triangular plane through Z_A, T_A, and T_A'. It is also possible to be a net user of Z in producing X, and the maximum amount of X that can be produced that way is graphed together with the equal negative input of Z at point Z_A' (which is on the dotted 45^o line in the horizontal (X,$-Z$) plane). Again, linear technologies also permit production anywhere on the triangular plane through Z_A', T_A, and T_A'. If fragmentation were costly, the surface combining these two triangles would be kinked along T_AT_A' (i.e., the straight line connecting Z_A and Z_A' would pass to the left of T_A'). But with costless fragmentation as I assume here, the two triangles are part of the same plane, and the full PPF of country A with fragmentation consist of the larger triangle $T_AZ_AZ_A$'. To assure that Country A will have a comparative advantage in X both with and without fragmentation, I have drawn these production possibilities as skewed in the X direction.

The top right portion of Figure 2 shows the PPF for Country B, with its greatest productivity being in the directions of final good Y and intermediate good Z. Otherwise the graph for Country B has exactly the same interpretation as for Country A.

Now combine these two PPFs, to get the PPF of the world in the bottom part of the Figure. The construction is a little messy but actually not all that difficult if you draw it carefully. As is familiar in the 2-good Ricardian model, one simply places the origin for one PPF on the surface of the other, then slides it around in all possible ways to generate the largest outputs possible. In two dimensions only, this process of adding together T_AT_A' and T_BT_B' would yield the world frontier T_WT_W'T_W" that appears in the X-

Y plane in the bottom graph. It is linear with just one kink (at T_W'), and the two linear segments are exact replicas of the transformation curves of the two individual countries.

To construct the PPF in three dimensions, I place the origin of Country A's PPF (shown as a shaded entity drawn above the large equal sign) on the surface of Country B's PPF, $T_BZ_BZ_B$'. As a result of the shapes to the two PPFs, point Z_A' of A's PPF traces out a duplicate of B's PPF: the triangle $Z_W^{"}Z_W^{'''}Z_W^{''''}$. The bottom edge of A's PPF, Z_AZ_A' traces out the parallelogram $Z_WZ_W^{'}Z_W^{"}Z_W^{''''}$ as its origin traverses T_BZ_B. And A's PPF itself becomes the topmost facet of the world PPF, the triangle $T_WZ_W^{'}Z_W^{"}$. Note that the parameters underlying these figures are such that only three patterns of specialization are possible: County B produces only good *Y* (the top triangle); Country A produces only good *X* using the fragmented technology and imported inputs of good *Z* (the lower triangle); and Country B produces a mix of *Y* and *Z* while Country A produces only *X*, mixing the original and the fragmented technology (the parallelogram).

However, the world cannot be in equilibrium just anywhere on the three dimensional surface. By assumption, good *Z* has no use except in producing *X*, so the world's net output of *Z* must be zero. Therefore the only relevant part of the world PPF is the solid heavy kinked line, $T_WT_W^{'}T_W^{'''}T_W^{''''}$. Just where on this curve the equilibrium will be found depends on preferences of consumers in both countries for consuming goods *X* and *Y*.

The potential gains for the world from fragmentation are now evident. Without fragmentation, the world was confined to the output combinations on T_WT_W'T_W", and if preferences put the equilibrium on the steeper segment, T_W'T_W", then fragmentation

permits an increase in the world consumption of both goods. These gains should not be surprising: they are simply the result of comparative advantage being followed within the fragmented technology for producing good *X*.

What are prices in the world equilibrium? They are given by the slopes of a plane tangent to the world PPF wherever the equilibrium is located. As usual in a Ricardian model, these may be determined mostly by preferences if the equilibrium is at one of the kinks, $T_W^{'}$ or $T_W^{'''}$, or they may be determined entirely by parameters of production if the equilibrium is in one of the straight segments. Note that if demand for *X* is sufficiently high so that the equilibrium is in the steeper of these segments, $T_W^{'''}T_W^{''''}$, then prices are those of Country B in autarky and do not change with fragmentation. That is the situation addressed in our analysis of the small country above, thus indicating that the assumptions made there were appropriate. Of course in this case, represented for example at point *I* in Figure 2, Country A does not lose from fragmentation, as already noted.

The possibility of loss arises in the other segment, $T_W^{'}T_W^{'''}$, at points like the one marked *II*. Here the relative price of good *X* falls as a result of fragmentation, and since Country A initially exported *X*, this constitutes a fall in its terms of trade. This is not enough to show that it loses, however, since it would have gained a positive amount from the fragmentation itself had prices not fallen. It turns out, though, that Country A *must* lose from fragmentation in this case. Segment $T_W^{'}T_W^{'''}$ is part of the parallelogram traced out by the lower edge of Country A's PPF as its origin traverses the upper left edge of B's PPF. Therefore the price plane tangent to the world PPF in this segment includes the entire bottom edge of A's PPF. This is enough to assure that there are no gains to it from

shifting resources toward fragmented production, and the reduced terms of trade therefore causes an unambiguous loss.

Both results are illustrated in Figure 3. In the top panel, prices remain fixed with fragmentation and country A gains unambiguously. Initially trading only X and Y , it consumed at point C along the two-dimensional price line given it by Country B. With fragmentation prices do not change, but Country A shifts all of its labor to producing good X with inputs of Z and the fragmented technology. Production moves to point Z_A' and it trades from there to point C' along the price plane, also given it by Country B. The nature of the trade, incidentally, is that it imports both Y and Z, since it produces neither.

The other case is shown in the bottom part of the figure. Here again Country A starts at point C, but now with fragmentation it trades along a price plane that is both flatter (lower p_X/p_Y) and rotated clockwise from above (lower p_X/p_Z), as shown. The loss in welfare is unambiguous. Production, incidentally, is somewhere on the bottom edge between points T_A' and Z_A', and the country imports Y and Z. The main difference between this and the case above is that the country also produces some of good X from scratch, and indeed it is this that requires prices that make the two processes for producing X equivalent and prevents any gain to Country A from using the fragmented technology. Country B, of course, now does gain.

III. Fragmentation in Heckscher-Ohlin

The simplicity of the Ricardian model is valuable for the insights that it can yield into the behavior of more complicated models. For example, a Heckscher-Ohlin (H-O) model can approximate arbitrarily closely to the Ricardian model of Section II, and

therefore we can conclude immediately that a large country can lose from fragmentation, even in a H-O world. However, there are other details that are assumed away in a Ricardian model that it therefore cannot address. The most obvious and important is the existence of separate factors of production and the possibility that some factors may gain while others may lose from a change such as fragmentation. I will explore that possibility here.

The first thing to note about a H-O model, however, is that fragmentation as defined here will not occur in the kinds of equilibria most often considered in the H-O literature. That is, if prior to fragmentation there is factor price equalization (FPE), and if fragmentation is costly, then there will be no incentive to fragment production. Therefore, in order for the issue of fragmentation to be interesting, we must start with a world economy in which factor prices are different. That is, I will assume that factor endowments differ sufficiently across countries that they are unable, in free trade, to produce enough goods in common to cause FPE. I will also assume, for simplicity, that fragmentation is costless, although one could easily think of there existing an infinitesimal fragmentation cost, ε, that is too small to show up in the diagrams. Again, I will start with a small country and then move to a 2-country model.

A Small Open H-O Economy

Figure 4 shows what can happen in a 2-good model. The solid lines show the initial situation before fragmentation. It is a familiar Lerner-Pearce diagram with good *X* chosen as numeraire so that that the unit value isoquant for *X* is also its unit isoquant.

A small country trades with a large rest-of-world in which prices of the two goods, *X* and *Y*, are given and imply the unit value isoquants shown. The diversification cone is given by the rays k_X and k_Y, with corresponding factor prices $\tilde{w}$ and $\tilde{r}$. The small country, however, is not inside the cone. Its endowment point is *E*, and it therefore specializes in good *X* with factor prices w_0 and r_0.

Now fragmentation becomes possible everywhere. The technology for producing good *X* now includes the possibility of producing an intermediate input, *Z*, one unit of which requires the factors shown by the new isoquant labeled Z=1.[8] A unit of good *Z* can be used together with additional inputs of capital and labor to produce a unit of *X*. Since I am now assuming that fragmentation is costless, the isoquant for producing a unit of *X* from *Z* is simply the factors that are left over out of the original X=1 isoquant. That is, the isoquant for producing *X* from *Z* can be drawn upside down relative to, say, point O_{XZ} as an origin, and it will then be just tangent to the Z=1 isoquant.[9]

Note that I have drawn the technology for producing *Z* as more capital intensive than for producing *X*. With that assumption, the activities available to the world now include one whose factor requirements (at the rest-of-world's factor prices) are given by the ray k_Z. The diversification cone is therefore enlarged by fragmentation. In the case shown, the small country is now inside it, and it will therefore now be able to fully employ its factors at the world's factor prices. This is now a 3-good, 2-factor H-O model,

[8] Again, this intermediate input need not be a physical entity, and therefore could represent, say, the "headquarter services" that Helpman (1984) and others have used to model multinational corporations. For my purposes, it is immaterial (literally) whether the different fragments of production are performed within a single firm or between firms. This generality is desirable, of course, but it also means that none of the results of this section are really new. They are merely extensions of what was shown by Helpman (1984) and Helpman and Krugman (1985). I am grateful to Steve Matusz for pointing this out.

so exact outputs and the pattern of trade are indeterminate *a la* Melvin (1968). But we can be sure in this case that the small country will produce some of intermediate good Z together with some of either X and/or Y.

More important here, however, is what happens to factor prices. Evidently from the figure, the wage has fallen from w_0 to $\tilde{w}$, while the rental on capital has risen from r_0 to $\tilde{r}$. Since prices are fixed, these changes are real as well as nominal.

Thus we have a case here where a country gains from fragmentation,[10] but not all factor owners within that country share in the gain. On the contrary, workers here are made worse off as the production process for their product fragments into a capital-intensive and a labor-intensive part, only the former of which necessarily remains viable within their country.

A Two Country H-O World

What happened in Figure 4 did not really require that the country in question be small. To see this, consider a two-country world of the same sort. Figure 5 shows the Travis-Dixit-Norman[11] integrated world economy (IWE) diagram that Helpman and Krugman (1985) exploited so successfully for a variety of purposes. Without

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[9] If these were drawn correctly, then the X=1 isoquant could be traced out by the origin for the XZ=1 isoquant as it slides around the Z=1 isoquant. The construction is much the same as we used to use to construct Scitovsky indifference curves. Remember them?

[10] I have not shown that gain, but I hope it is clear. Fragmentation has expanded the small country's production possibilities, and at fixed prices this can only help.

[11] Years ago, I incorrectly attributed this diagram to Helpman and Krugman in the presence of both Elhanan Helpman and Avinash Dixit and had my attention gently directed toward Dixit and Norman (1980). More recently, Bob Baldwin has sent me further back, to Travis (1964). While Travis did not speak by that name of an "integrated world economy," he certainly used the concept and drew the diagram (on p. 16) for exactly the purpose of identifying a factor price equalization region. Travis in turn called it an Edgeworth box, although I am not aware (never having read Edgeworth) that Edgeworth ever used his boxes quite this way, with factor endowments measured from the corners. All of this is surely known to scholars, but the rest of us have to pick it up wherever we can.

fragmentation, the FPE region is the parallelogram O_AFO_BF'. Fragmentation, which makes it possible to duplicate the *X* output of factor vector O_AF with the two fragmented vectors O_AG and *GF*, expands the FPE region to $O_AGFO_BG'F'$. Thus, for some allocations of the world's factors for which FPE would not have been possible, fragmentation leads to FPE.

For example, if the allocation were at point *E*, then there could not have been FPE without fragmentation. What exactly would have happened instead we don't know, since prices would not have been those prevailing in the IWE, but it seems plausible that Country A would have produced only good *X* and Country B would have produced a mix of *X* and *Y*.[12] Fragmentation again permits Country A to shift resources into the most capital-intensive fragment of producing *Z*, and again it is likely to raise the return to capital and lower the wage. There may also be small effects on factor prices in Country B, as market clearing prices of *X* and *Y* change, but these are difficult to determine.

This result illustrates the more general phenomenon that fragmentation enhances the possibility of factor price equalization. With arbitrary numbers of goods, factors, and countries, I showed in Deardorff (1994) that FPE requires that factor endowments be more similar than factor intensities in a certain well-defined sense: the set of vectors that can be constructed from parts (or all) of the factor endowment vectors – called the "factor endowment lens" – must lie inside the analogous set of vectors defined by the factors used in the IWE – the "factor use lens." Costless fragmentation unambiguously enlarges

[12] If we assume fixed-coefficient technologies, then this is necessarily the case, since good and factor prices will leave the factor use vectors unchanged. Of course in that case, part of the capital in Country A of Figure 5 would be unemployed and the rental on capital would be zero. However the changes in factor prices identified in Figure 4 would still occur, in amplified form.

the factor use lens and thus makes FPE possible for factor allocations for which it was not possible before.

To see this, note that a factor use lens is defined as

$$\mathcal{L}(v) = \{x \in \mathbb{R}^f \,|\, x = bv \text{ for some } g\text{-vector } b \in I^g\} \tag{6}$$

where there are f factors and g goods, I^g is a set of g-vectors whose elements are in the unit interval, [0,1], and v is a $g \times f$ matrix of factor-use row vectors. Then let v^0 be the matrix of factor uses without fragmentation, v_j^0, j=1,…,g, and let v^1 be the matrix of factor uses in which some or all goods have been fragmented into 2 or more parts. Numbering the rows of v^1 ji, i=1,…,n_j, the cost of fragmentation will be zero if $v_j^0 = \sum_{i=1}^{n_j} v_{ji}^1$. It is then easily shown that[13,14]

$$\mathcal{L}(v^1) \subseteq \mathcal{L}(v^0) \tag{7}$$

This tells us that the possibility of FPE is increased by fragmentation. That is, there may be allocations of the word's factor endowments for which FPE is not possible without fragmentation and for which FPE becomes possible with fragmentation. It also says that if FPE is possible without fragmentation, then it must remain possible if fragmentation is introduced. If one were comfortable thinking of the world's allocations of factors as being somehow randomly determined given its total endowments and the parameters of technology and tastes, then it would say that the likelihood of FPE either rises or stays the same with fragmentation; it cannot fall.

[13] Suppose $x \in \mathcal{L}(v^0)$. Then from (6) $\exists b \ni x = bv^0 = b\Sigma v_i^1$ which implies $x \in \mathcal{L}(v^1)$.

[14] See Debaere and Demiroglu (1997), who note the same thing in a different context. Their result is that the measured factor use lens will lie within the true one if measured industries are aggregated.

Thus fragmentation may well cause factor prices to become equalized across countries. The analysis here does *not* say, however, that, failing that, factor prices will be drawn closer together by fragmentation. Figure 4 certainly suggests that result in a special case. There, if the endowment point were above the k_Z ray rather than below it as drawn, then indeed factor prices in the country would remain different from those in the world after fragmentation but would be drawn closer to them.[15] As far as I know, however, that is not a general result. In fact, we are surprisingly ignorant (or at least I am) of how the higher-dimensional H-O model behaves outside of the FPE region, or when the lens condition of Deardorff (1994) is not satisfied.

IV. Conclusion

The underlying question addressed in this paper is, "Does fragmentation matter?" Or, since this is a theoretical piece, not empirical, "Can it matter?" The answer seems to be a relatively strong "Yes."

Of course, fragmentation will *not* matter if factor prices are equal everywhere, for then the fragmented technologies will at best duplicate what was done without them and there will be no reason to use them. But if factor prices are not equal across countries, either because technologies differ as in the Ricardian model or because factor endowments are less similar than factor intensities in the H-O model, then fragmentation can certainly make a difference. The main effects of fragmentation, at least as I have

[15] Such a result may exist, but I am pessimistic. After wasting several years trying to show that free trade must draw factor prices closer together, only to find the counterexample presented in Deardorff (1986), I will not be surprised if fragmentation admits of equally poorly behaved possibilities in a general model.

been able to identify them from these simple models, are as follows (all assuming that fragmented technologies are used at all):

1. If fragmentation does not change the prices of goods, then it must increase the value of output of any country where it occurs and that of the world.

2. If fragmentation does change prices, then fragmentation can lower the welfare of a country by turning its terms of trade against it.

3. Even in a country that gains from fragmentation, it is possible (but not necessary) that some factor owners within that country will lose.

4. To the extent that factor prices are not equalized internationally without fragmentation, fragmentation may be a force toward factor price equalization.

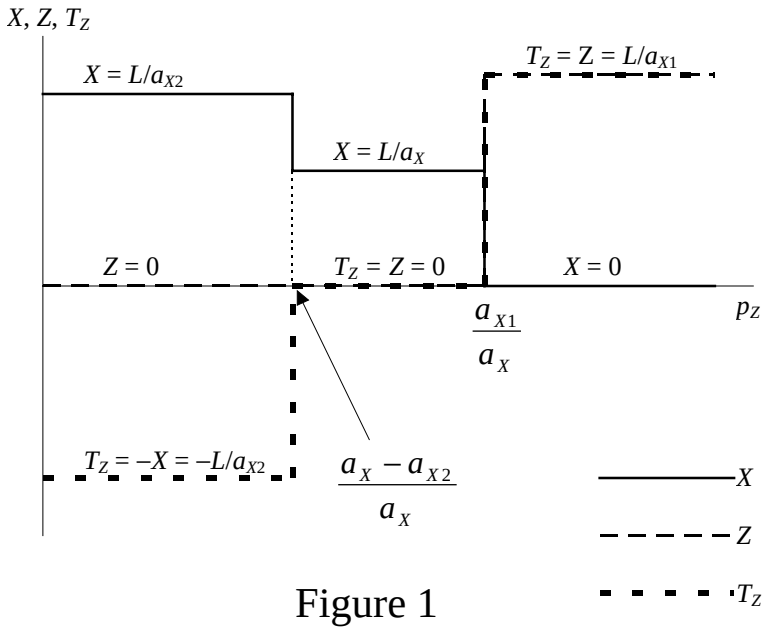
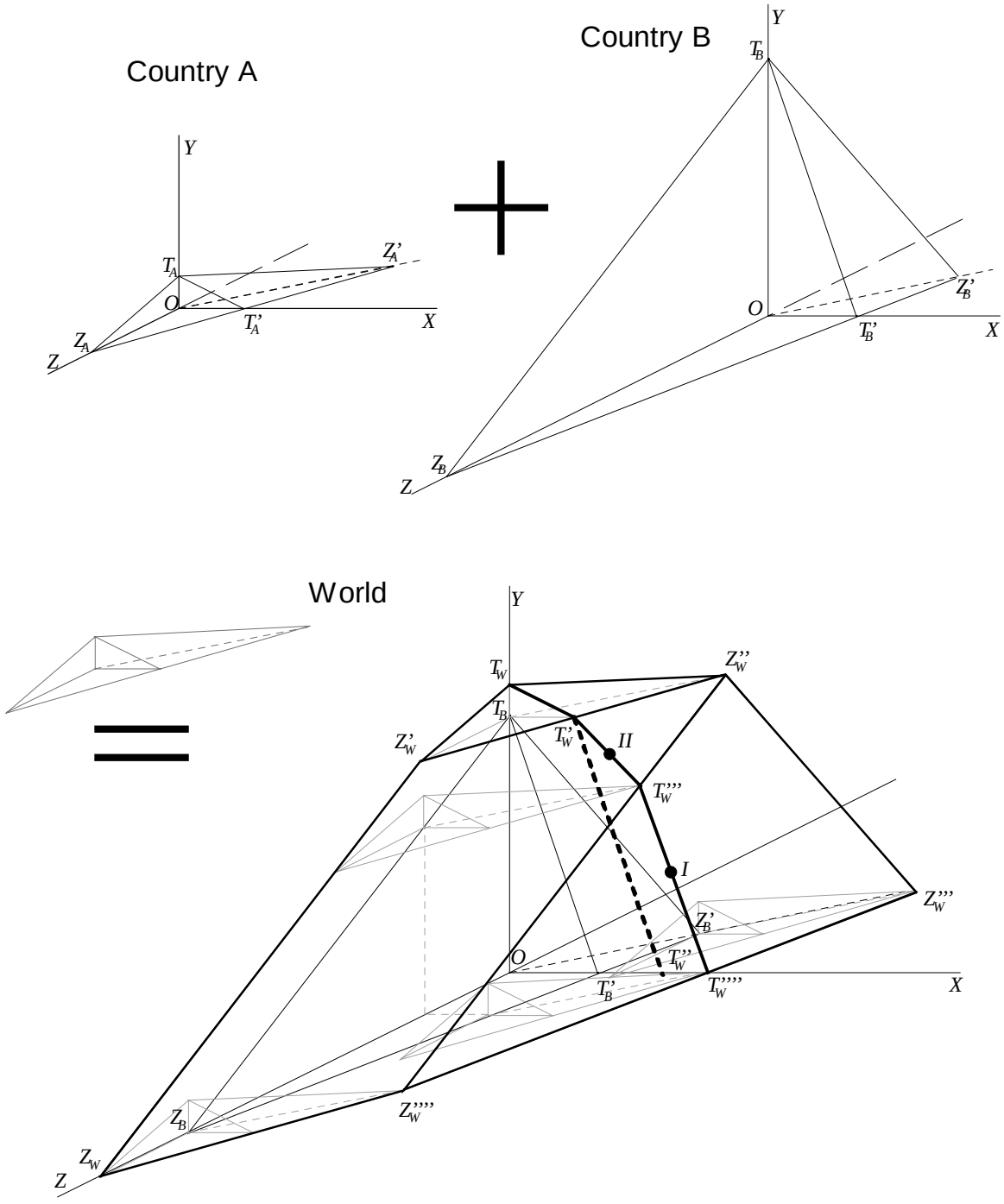

X, Z, T_Z
X = L/a_X2
T_Z = Z = L/a_X1
X = L/a_X
Z = 0
T_Z = Z = 0
X = 0
p_Z
a_X1 / a_X
T_Z = –X = –L/a_X2
a_X – a_X2 / a_X
X
Z
T_Z

Figure 1


Country A
Country B
World
Y
X
Z
O
T_A
T_A'
Z_A
Z_A'
T_B
T_B'
Z_B
Z_B'
T_W
T_W'
T_W''
T_W'''
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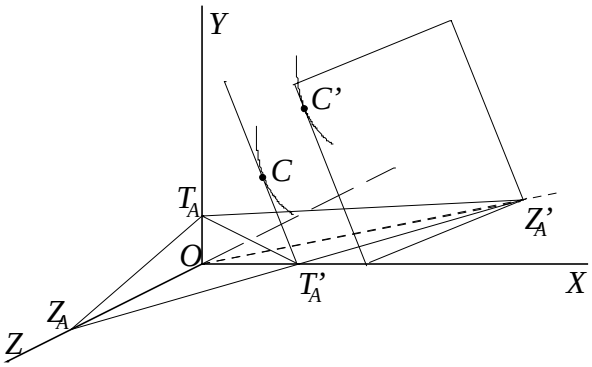
I
II

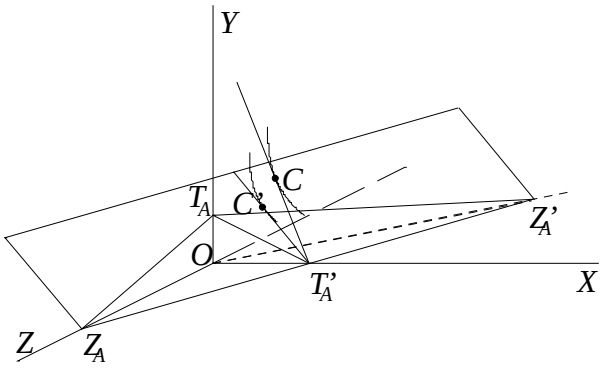
Figure 2

Country A



3-1: Equilibrium at *I* in Figure 2

Country A



3-2: Equilibrium at *II* in Figure 2

Figure 3

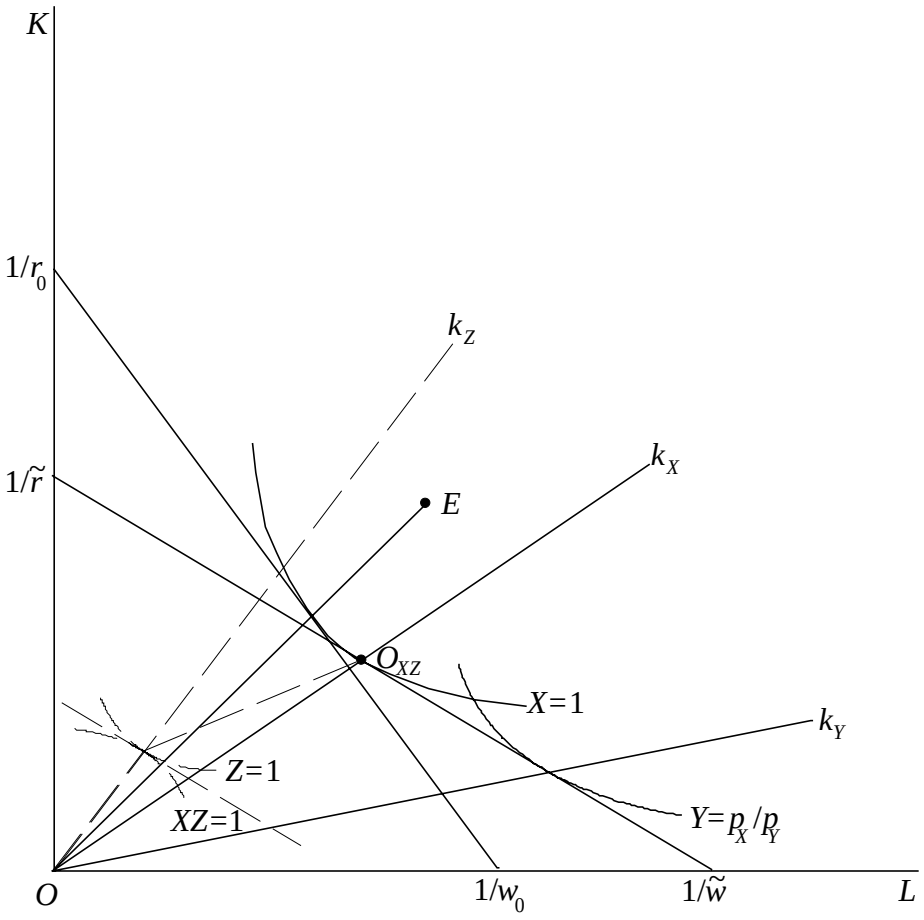


Figure 4

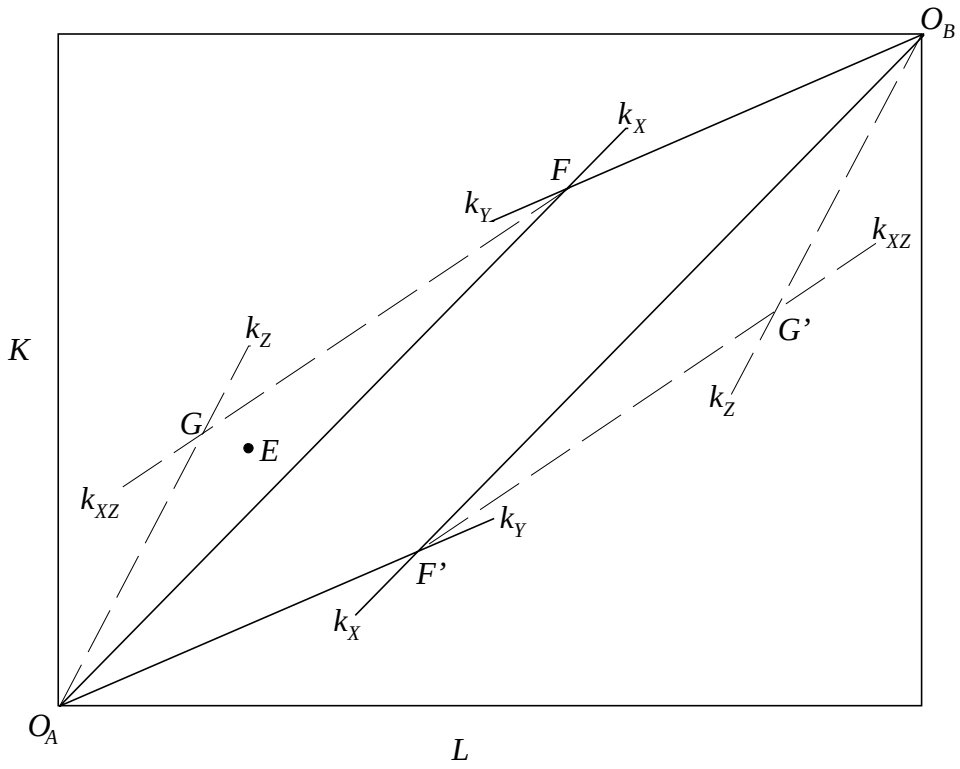


Figure 5

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