DERIVATIONS OF EQUATIONS FOR THE MICHIGAN BROWN-DEARDORFF-STERN CGE TRADE MODEL

Country Equations (i=1,…,m ;j=1,…,n)

A. Final Demand

Consumers are taken to use a three-stage process for allocating expenditure across competing suppliers. At the first stage, expenditure is allocated across the n different broad product categories assuming a Cobb-Douglas utility function. Therefore, they are taken to

$$(A1)\qquad \underset{\{C_{i1},\ldots,C_{in}\}}{\text{Max}} \quad U_i = \prod_{j=1}^{n} C_{ij}^{a_{ij}} \quad \text{s.t.} \quad \sum_{j=1}^{n} P_{ij} C_{ij} = E_i$$

which yields the following demand equations

$$(A2)\qquad C_{ij} = \frac{a_{ij} E_i}{P_{ij}} .$$

Differentiating (A2) yields

$$(A3)\qquad \hat{C}_{ij} = \hat{E}_i - \hat{P}_{ij}$$

where the circumflex indicates proportionate change.

B. Total Demand

Total demand is composed of intermediate and final demand. Final demand is derived above. To find intermediate demand, producers are assumed to use intermediate input aggregates in fixed proportion to production. Therefore intermediate demand for good j used in industry k is proportional to output in industry k. That is

$$(B1)\qquad \hat{Z}_{ijk} = \hat{S}_{ik} .$$

The proportionate change in total demand is simply the demand share weighted average of final and intermediate demands. That is

$$(B2)\qquad \hat{D}_{ij} = v_{ijo} \hat{C}_{ij} + \sum_{k=1}^{n} v_{ijk} \hat{S}_{ik} .$$

C. Product Demand

Demands for the output of individual firms are generated in the second and third decision stages. In the second stage users are assumed to allocate expenditure between domestic varieties and an import aggregate. The aggregation function at this stage is given by

$$(C1) \qquad D_{ij} = [n_{ij}^{1+m}(D_{ij}^{i})^{r} + (D_{ij}^{M})^{r}]^{1/r}$$

where : is a measure of the value that consumers place on different varieties of the domestic good. The corresponding maximization problem is given by

$$(C2) \qquad \underset{\{D_{ij}^{i}, D_{ij}^{M}\}}{Max} \quad n_{ij}^{1+m}(D_{ij}^{i})^{r} + (D_{ij}^{M})^{r} + l[E_{ij} - n_{ij}D_{ij}^{i}P_{ij}^{i} - D_{ij}^{M}P_{ij}^{M}].$$

The first order conditions for this problem are

$$(C3) \qquad r n_{ij}^{m}(D_{ij}^{i})^{r-1} - lP_{ij}^{i} = 0 \qquad and \qquad r(D_{ij}^{M})^{r-1} - lP_{ij}^{M} = 0.$$

If we solve each of the first order conditions for their respective demands and substitute into the constraint, we can then eliminate the lagrange multiplier. This yields the following demands

$$(C4) \qquad D_{ij}^{i} = \frac{E_{ij}(P_{ij}^{i})^{-s}\, n_{ij}^{ms}}{n_{ij}^{1+ms}(P_{ij}^{i})^{1-s} + (P_{ij}^{M})^{1-s}} \qquad and \qquad D_{ij}^{M} = \frac{E_{ij}(P_{ij}^{M})^{-s}}{n_{ij}^{1+ms}(P_{ij}^{i})^{1-s} + (P_{ij}^{M})^{1-s}}.$$

Proportionately differentiating the demands given by (C4) yields

$$(C5) \qquad \hat{D}_{ij}^{i} = \hat{E}_{ij} - s\hat{P}_{ij}^{i} + ms\hat{n}_{ij} - (1+ms)q_{ij}^{i}\hat{n}_{ij} + (s-1)(q_{ij}^{i}\hat{P}_{ij}^{i} + q_{ij}^{M}\hat{P}_{ij}^{M})$$

and

$$(C6) \qquad \hat{D}_{ij}^{M} = \hat{E}_{ij} - s\hat{P}_{ij}^{M} - (1+ms)q_{ij}^{i}\hat{n}_{ij} + (s-1)(q_{ij}^{i}\hat{P}_{ij}^{i} + q_{ij}^{M}\hat{P}_{ij}^{M})$$

where we have made use of the fact that the budget shares can be shown to be

$$(C7) \qquad q_{ij}^{i} = \frac{n_{ij}P_{ij}^{i}D_{ij}^{i}}{E_{ij}} = \frac{n_{ij}^{1+ms}(P_{ij}^{i})^{1-s}}{n_{ij}^{1+ms}(P_{ij}^{i})^{1-s} + (P_{ij}^{M})^{1-s}}$$

and

$$(C8) \qquad q_{ij}^{M} = \frac{P_{ij}^{M}D_{ij}^{M}}{E_{ij}} = \frac{(P_{ij}^{M})^{1-s}}{n_{ij}^{1+ms}(P_{ij}^{i})^{1-s} + (P_{ij}^{M})^{1-s}}$$

and the elasticity of substitution between the import aggregate and varieties of the domestic good is given by

$$s = \frac{1}{1-r}.$$

Expenditure on good j in equations (C5) and (C6) can be decomposed into a goods index and a price index. To derive these indexes first note that expenditure in the above equations must equal expenditure over the various varieties of good j. That is

$$(C9)\qquad E_{ij} = P_{ij} D_{ij} = n_{ij} P^i_{ij} D^i_{ij} + P^M_{ij} D^M_{ij}.$$

Proportionate differentiation yields

$$(C10)\qquad \hat{E}_{ij} = \hat{P}_{ij} + \hat{D}_{ij} = (\hat{n}_{ij} + \hat{P}^i_{ij} + \hat{D}^i_{ij}) q^i_{ij} + (\hat{P}^M_{ij} + \hat{D}^M_{ij}) q^M_{ij}.$$

The goods aggregator can easily be found by proportionately differentiating equation (C1) above to obtain

$$(C11)\qquad \hat{D}_{ij} = q^i_{ij} \hat{D}^i_{ij} + q^M_{ij} \hat{D}^M_{ij} + \frac{(1+m)s}{s-1} q^i_{ij} \hat{n}_{ij}$$

where we have made use of the fact that the market shares can be also represented by

$$(C12)\qquad q^i_{ij} = \frac{n^{1+m}_{ij} (D^i_{ij})^r}{n^{1+m}_{ij} (D^i_{ij})^r + (D^M_{ij})^r} \qquad and \qquad q^M_{ij} = \frac{(D^M_{ij})^r}{n^{1+m}_{ij} (D^i_{ij})^r + (D^M_{ij})^r}.$$

Substituting equation (C11) into (C10) allows us to calculate the price index

$$(C13)\qquad \hat{P}_{ij} = q^i_{ij} \hat{P}^i_{ij} + q^M_{ij} \hat{P}^M_{ij} - \frac{(1+ms)}{s-1} q^i_{ij} \hat{n}_{ij}.$$

The price index can then be substituted into the demand equations (C5) and (C6) to obtain

$$(C5')\qquad \hat{D}^i_{ij} = \hat{D}_{ij} + s q^M_{ij} (\hat{P}^M_{ij} - \hat{P}^i_{ij}) - \frac{s(1+ms)}{s-1} q^i_{ij} \hat{n}_{ij} + ms\hat{n}_{ij}$$

$$(C6')\qquad \hat{D}^M_{ij} = \hat{D}_{ij} + s q^i_{ij} (\hat{P}^i_{ij} - \hat{P}^M_{ij}) - \frac{s(1+ms)}{s-1} q^i_{ij} \hat{n}_{ij}.$$

Once consumers have allocated expenditure between domestic varieties and the import aggregate, expenditure on imports is then allocated across foreign suppliers. The import aggregator is

$$(C14)\qquad D^M_{ij} = [\sum_{r \neq i}^{m+1} n^{1+m}_{rj} (D^r_{ij})^r]^{1/r}$$

which is maximized subject to the condition that

$$(C15) \quad E_{ij}^{M} = P_{ij}^{M} D_{ij}^{M} = \sum_{r \neq i}^{m+1} n_{rj} P_{ij}^{r} D_{ij}^{r}.$$

As demonstrated above, the corresponding demand equations are given by

$$(C16) \quad D_{ij}^{r} = \frac{E_{ij}^{M} (P_{ij}^{r})^{-s} n_{rj}^{ms}}{\sum_{s \neq i}^{m+1} n_{sj}^{1+ms} (P_{ij}^{s})^{1-s}} \qquad r \neq i; r = 1,\ldots,m+1$$

which when proportionately differentiated becomes

$$(C17) \quad \hat{D}_{ij}^{r} = \hat{E}_{ij}^{M} - s\hat{P}_{ij}^{r} + ms\hat{n}_{rj} - \sum_{s \neq i}^{m+1} [(1+ms)\hat{n}_{sj} + (1-s)\hat{P}_{ij}^{s}] q_{ij}^{sM}.$$

Also as above, we can find that the import price index is

$$(C18) \quad \hat{P}_{ij}^{M} = \sum_{r \neq i}^{m+1} q_{ij}^{rM} \hat{P}_{ij}^{r} - \sum_{r \neq i}^{m+1} \frac{(1+ms)}{s-1} q_{ij}^{rM} \hat{n}_{rj}.$$

Therefore, equation (C17) can be rewritten as

$$(C17') \quad \hat{D}_{ij}^{r} = \hat{D}_{ij}^{M} + s(\hat{P}_{ij}^{M} - \hat{P}_{ij}^{r}) + ms\hat{n}_{rj}$$

simply by noting that the last summation on the right hand side of (C17) is simply the import price index multiplied by (σ-1).

D. Prices

The landed price of imports of good j from country r paid by the consumer is given by the world price times one plus the ad valorem tariff. That is

$$(D1) \quad P_{ij}^{r} = P_{wj}^{r}(1 + t_{ij}^{rMeq}) \qquad r \neq i;\ r = 1,\ldots,m+1$$

which when proportionately differentiated becomes

$$(D2) \quad \hat{P}_{ij}^{r} = \hat{P}_{wj}^{r} + \hat{t}_{ij}^{rMeq} \qquad r \neq i;\ r = 1,\ldots,m+1.$$

The price received by the producer must satisfy two conditions. First, free entry guarantees that profits are zero. We assume that the producer has a fixed cost of capital and labor and then constant marginal cost. Therefore, price must equal marginal cost plus average fixed cost:

$$(D3) \qquad P_{wj}^{i} = MC_{ij} + \frac{P_{ij}^{V}}{S_{ij} / n_{ij}}.$$

P^V is a price index for the primary inputs needed for both fixed and variable costs, as will be disucussed below and S/n is output per firm. When equation (D3) is proportionately differentiated it becomes

$$(D4) \qquad \hat{P}_{wj}^{i} = q_{ij}^{MC} M\hat{C}_{ij} + q_{ij}^{FC} (\hat{P}_{ij}^{V} + \hat{n}_{ij} - \hat{S}_{ij}).$$

Firms are also taken to set price as a profit-maximizing mark-up over marginal cost. Following the standard monoply pricing rule we have

$$(D5) \qquad P_{wj}^{i} = \frac{MC_{ij}}{1 + 1/h_{ij}}$$

which when proportionately differentiated becomes

$$(D6) \qquad \hat{P}_{wj}^{i} = M\hat{C}_{ij} + \frac{\hat{h}_{ij}}{1 + h_{ij}}.$$

E. Marginal Cost

Margainal cost can be found by summing over the cost of all the variable inputs. That is

$$(E1) \qquad MC_{ij} = a_{ij0} P_{ij}^{V} + \sum_{k=1}^{n} a_{ikj} P_{ik}$$

where a_{ikj} is the unit input requirement of good k in industry j and a_{ijo} is the variable unit input requirement for the primary input aggregate. (The primary input aggregate must be divided input fixed and variable because each firm is assumed to require some fixed input of capital and labor.) Proportionate differentiation of (E1) gives

$$(E2) \qquad M\hat{C}_{ij} = \frac{a_{ij0} P_{ij}^{V}}{MC_{ij}} \hat{P}_{ij}^{V} + \sum_{k=1}^{n} \frac{a_{ikj} P_{ik}}{MC_{ij}} \hat{P}_{ik}.$$

Equation (E2) can be rewritten as

$$(E2') \qquad M\hat{C}_{ij} = \frac{q_{ij}^{VK}}{q_{ij}^{MC}} b_{ijo} \hat{P}_{ij}^{V} + \sum_{k=1}^{n} \frac{b_{ikj}}{q_{ij}^{MC}} \hat{P}_{ik}$$

where b_{ikj} is input k's share of total cost in industry j, b_{ijo} is the primary input aggregates share of total cost in industry j and θ^{MC}=MC/ATC is marginal cost's share of total cost and θ^{VK} is the proportion of the primary input aggregate that is variable (as opposed to fixed).

F. Primary Factor Demands

Primary inputs, capital and labor, are also aggregated using a CES aggregation function. A unit of the primary input aggregate is given by

$$(F1) \qquad V = [K^{r} + L^{r}]^{1/r}.$$

In order to find contional factor demands total cost, given by

$$(F2) \qquad TC = wL + rK,$$

is minimized subject to the constraint in (F1). The first order conditions for this minimization problem are

$$(F3)\ w = \ lrL^{r-1} \qquad and \qquad r = \ lrK^{r-1}.$$

If we solve the first order conditions for K and L to get

$$(F3') \qquad L = [\frac{w}{lr}]^{\frac{1}{r-1}} \qquad and \qquad K = [\frac{r}{lr}]^{\frac{1}{r-1}}$$

and substitute into the total cost equation (F2) we obtain

$$(F2') \qquad TC = [w^{\frac{r}{r-1}} + r^{\frac{r}{r-1}}](\ lr)^{\frac{1}{1-r}}.$$

Equation (F2') can then be solved for the λρ term. When substituted into equation (F3') we obtain

$$(F3'') \qquad L = \frac{v^{\frac{1}{r-1}}TC}{w^{\frac{r}{r-1}} + r^{\frac{r}{r-1}}} \qquad and \qquad K = \frac{w^{\frac{1}{r-1}}TC}{w^{\frac{r}{r-1}} + r^{\frac{r}{r-1}}}.$$

If we now substitute K and L from equaiton (F3'') into the aggregation function (F1) we find that

$$(F4) \qquad V = \frac{TC}{[w^{\frac{r}{r-1}} + r^{\frac{r}{r-1}}]^{\frac{r-1}{r}}}.$$

Inverting equation (F4) gives us the total cost function:

$$(F2'') \qquad TC = \frac{V}{[w^{\frac{r}{r-1}} + r^{\frac{r}{r-1}}]^{\frac{1-r}{r}}}.$$

Finally, we apply Shephard's lemma to obtain the conditional factor demands:

$$(F3'') \quad L = \frac{Vw^{-\bar{s}}}{[w^{1-\bar{s}} + r^{1-\bar{s}}]^{\frac{\bar{s}}{1-\bar{s}}}} \qquad and \qquad K = \frac{Vr^{-\bar{s}}}{[w^{1-\bar{s}} + r^{1-\bar{s}}]^{\frac{\bar{s}}{1-\bar{s}}}}.$$

Proportionately differentiating equation (F3'') gives

$$(F3''') \qquad \hat{L} = \hat{V} + \bar{s}q^{K}(\hat{r} - \hat{w}) \qquad and \qquad \hat{K} = \hat{V} - \bar{s}q^{L}(\hat{r} - \hat{w}).$$

We have in equation (F3''') firm demand for labor and capital as a function of demand for the primary input aggregate (V) and factor returns (w and r). For the purposes of the model, however, we need industry demand for labor and capital. The industry demand for each labor is

$$(F5) \qquad L_{ij} = n_{ij}(L_{ij}^{V} + L_{ij}^{F})$$

where V and F denote variable and fixed labor. Proportionate differentiation yields

$$(F6) \qquad \hat{L}_{ij} = \hat{n}_{ij} + q_{ij}^{VK}\hat{L}_{ij}^{V} + q_{ij}^{FK}\hat{L}_{ij}^{FK}.$$

An analagous equation applies for capital. The only difference between variable labor demand and fixed labor demand is that fixed labor demand is not dependant on output (V). Therefore, we can substitute equation (F3''') into (F6) to get

$$(F7) \qquad \hat{L}_{ij} = \hat{n}_{ij} + q_{ij}^{VK}\hat{V}_{ij} + \bar{s}q_{ij}^{K}(\hat{r}_i - \hat{w}_i)$$

and

$$(F8) \qquad \hat{K}_{ij} = \hat{n}_{ij} + q_{ij}^{VK}\hat{V}_{ij} - \bar{s}q_{ij}^{L}(\hat{r}_i - \hat{w}_i).$$

G. Demand Elasticities

The firm's perceived elasticity of demand in equation (D3) is a market-share weighted average of elasticities in each of its destination markets. In order to find the elasticity for each market we just return to the demand equations in Section C above.

Although we have set up the consumer's maximization problem in three stages, we have chosen to set the elasticity of substitution among import varieties equal to the elasticity of substitution between domestic varities and the import aggregate. As a result, we can substitute equation (C14) into equation

(C1) to obtain a single aggregation function over all varieties of the good. The associated demands can readily be found to be

$$(G1) \qquad D_{ij}^{r} = \frac{E_{ij}(P_{ij}^{r})^{-s} n_{rj}^{ms}}{\sum_{s=1}^{m+1} n_{sj}^{1+ms} (P_{ij}^{s})^{1-s}} \qquad r = 1,\ldots,m+1.$$

Differentiating equation (G1) with respect to price allows us to calculate the corresponding demand

$$(G2) \qquad \frac{\partial D_{ij}^{r}}{\partial P_{ij}^{r}} = \frac{-sE_{ij}(P_{ij}^{r})^{-(1+s)} n_{rj}^{ms}}{\sum_{s=1}^{m+1} n_{sj}^{1+ms} (P_{ij}^{s})^{1-s}} - \frac{(1-s)n_{rj}^{ms}(P_{ij}^{r})^{-s}}{[\sum_{s=1}^{m+1} n_{sj}^{1+ms} (P_{ij}^{s})^{1-s}]^{2}} \qquad r = 1,\ldots,m+1.$$

elasticity.

Converting equation (G2) into an elasticity gives

$$(G2') \qquad h_{ij}^{r} = \frac{P_{ij}^{r}}{D_{ij}^{r}} \frac{\partial D_{ij}^{r}}{\partial P_{ij}^{r}} = -s + (s-1) \frac{n_{rj}^{ms}(P_{ij}^{r})^{1-s}}{\sum_{s=1}^{m+1} n_{sj}^{1+ms} (P_{ij}^{s})^{1-s}} \qquad r = 1,\ldots,m+1.$$

Making use of equation (C7) and (C8), we can rewrite (G2') as

$$(G2'') \quad h_{ij}^{r} = -s + (s-1) \frac{P_{ij}^{r} D_{ij}^{r}}{P_{ij} D_{ij}} \qquad r = 1,\ldots,m.$$

Proportionately differentiating equation (G2'') gives us the demand elasticties for each market:

$$(G3) \qquad \hat{h}_{ij}^{r} = \frac{(s-1)q_{ij}^{r}}{h_{ij}^{r} n_{rj}} (\hat{P}_{ij}^{r} + \hat{D}_{ij}^{r} - \hat{P}_{ij} - \hat{D}_{ij}) \qquad r = 1,\ldots,m+1.$$

Each firm then takes its own demand elasticity to be an sales-share weighted average of the elasticities in each of its markets. That is

$$(G4) \qquad h_{ij} = \sum_{r=1}^{m+1} d_{ij}^{r} h_{rj}^{i}.$$

Differentiating equation (G4) and then converting to proportionate changes gives

$$(G5) \qquad \hat{h}_{ij} = \sum_{r=1}^{m+1} \frac{d_{ij}^{r} h_{rj}^{i}}{h_{ij}} \hat{h}_{rj}^{i}.$$

H. Primary Factors Market Equilibrium

Capital and labor markets are taken to be perfectly competitive with both factors freely mobile across sectors. Therefore, factor market equilibrium simply requires that the employment changes across sectors sum to zero. In percent change terms we have

$$(H1) \qquad \sum_{j=1}^{n} K_{ij}\hat{K}_{ij} = 0$$

and

$$(H2) \qquad \sum_{j=1}^{n} L_{ij}\hat{L}_{ij} = 0.$$

I. Nontariff Barriers

The model includes the possibility of having bilateral import quotas applying to some fraction of trade in each product category. A tariff-equivalent of the quota is computed endogenously in the model so as to hold the covered imports to the quota level. The tariff equilvalent then applying to each bilateral trade flow is a weighted average of the tariff equivalent of the quota and the nominal tariff where the weights are the fraction of trade covered by each policy instrument.

We calculate the tariff equivalent in the following manner. Firm level demand can be found by applying the price elasticity to the tariff equivalent, plus a set of other exogenous factors. That is

$$(I1) \qquad \hat{D}_{ij}^{r} = h_{ij}^{r}\hat{t}_{ij}^{rMeq} + l_{ij}^{r}.$$

Similarly, the quota is met when the tariff equivalent of the quota is applied to a price elasticity. That is

$$(I2) \qquad \hat{Q}_{ij}^{r} - \hat{n}_{rj} = h_{ij}^{r}\hat{t}_{ij}^{rQ} + l_{ij}^{r}.$$

Using Equation (I1) and (I2) to eliminate λ and then solving for the tariff equivalent of the quota gives

$$(I3) \qquad \hat{t}_{ij}^{rQ} = \hat{t}_{ij}^{rMeq} + \frac{\hat{Q}_{ij}^{r} - \hat{D}_{ij}^{r} - \hat{n}_{rj}}{h_{ij}^{r}}.$$

The tariff equivalent applying to all trade is a weighted average of the tariff equivalent of the quota applying to quota-governed trade and the nominal tariff applying to other trade. That is

$$(I4) \qquad \hat{t}_{ij}^{rMeq} = q_{ij}^{rQ}\hat{t}_{ij}^{rQ} + (1 - q_{ij}^{rQ})\hat{t}_{ij}^{r}.$$

Substituting equation (I3) into (I4) gives the tariff equivalent applying to all imports of j from country r. That is

$$(I4') \qquad \hat{t}_{ij}^{rMeq} = \hat{t}_{ij}^{r} + \frac{q_{ij}^{rQ}}{(1-q_{ij}^{rQ})h_{ij}^{r}}(\hat{Q}_{ij}^{r} - \hat{D}_{ij}^{r} - \hat{n}_{rj}).$$

Finally, we can see from equation (G2'') that as long as the number of firms is large, implying that each firm's market share is small, that the elasticity of demand can be approximated by -σ. Making this substitution gives

$$(I4'') \qquad \hat{t}_{ij}^{rMeq} = \hat{t}_{ij}^{r} - \frac{q_{ij}^{rQ}}{(1-q_{ij}^{rQ})s}(\hat{Q}_{ij}^{r} - \hat{D}_{ij}^{r} - \hat{n}_{rj}) \qquad r = 1,\ldots,m+1.$$

World Equations

J. Trade Balance and Income Determination

Income in this model is determined by the sum of payments to factors plus net borrowing in the base period data set. This is equivalent to holding the change in the trade balance equal to zero.

The trade balance is simply the difference between the value of exports and imports. That is

$$(J1) \qquad B_i^T = \sum_{j=1}^{n} n_{ij} P_{wj}^{i} \sum_{r \neq i}^{m+1} D_{rj}^{i} - \sum_{j=1}^{n} \sum_{r \neq i}^{m+1} n_{rj} P_{wj}^{r} D_{ij}^{r}$$

where ROW demand is discussed below. Proportionately differentiating (J1) gives

$$(J2) \qquad 0 = dB_i^T = \sum_{j=1}^{n} [X_{ij} \hat{P}_{wj}^{i} + \sum_{r \neq i}^{m} X_{ij}^{r} (\hat{D}_{rj}^{i} + \hat{n}_{ij}) + X_{ij}^{m+1} L_E] - \sum_{j=1}^{n} \sum_{r \neq i}^{m+1} M_{ij}^{r} (\hat{n}_{rj} + \hat{P}_{wj}^{r} + \hat{D}_{ij}^{r}) \qquad i = 1,\ldots,m.$$

K. Goods Market Equilibrium

Equilibrium in the goods market simply requires that the demand for each firm's goods equals supply. That is

$$(K1) \qquad \frac{S_{ij}}{n_{ij}} = \sum_{r=1}^{m+1} D_{rj}^{i}.$$

Proportionately differentiating (K1) gives

$$(K1') \qquad S_{ij}\hat{S}_{ij} = n_{ij} D^{i}_{ROWj} L_E + \sum_{r=1}^{m} n_{ij} D^{i}_{rj} (\hat{D}^{i}_{rj} + \hat{n}_{ij}) \qquad j = 1,\ldots,n \quad i = 1,\ldots,m.$$

L. ROW Goods Market Equilbrium

The market for goods produced by the rest of the world is handled in a somewhat *ad hoc* manner. The percent change in supply is found by applying a supply elasticity to the percent change in price:

$$(L1) \qquad dS_j^{ROW} = S_j^{ROW} e_j^{ROW} \hat{P}_{Wj}^{ROW} \qquad j = 1,\ldots,n.$$

The change in supply is then equated with the the change in demand to find equilibrium prices:

$$(L2) \qquad dS_j^{ROW} = \sum_{i=1}^{m} D_{ij}^{ROW} \hat{D}_{ij}^{ROW} \qquad j = 1,\ldots,n.$$

M. ROW Import Licensing

ROW import demand is governed by a licensing variable, L_E. L_E is the percent change in the demand for imports that can be financed by export sales. This constraint, proportionately differentiated, is

$$(M1) \qquad \sum_{j=1}^{n} (dS_j^{ROW} + S_j^{ROW} \hat{P}_{wj}^{ROW}) = L_E M_{ROW} + \sum_{i=1}^{m} M^{i}_{ROWj} \hat{P}^{i}_{wj}.$$